Mathematical Modeling and Prediction of Road Accident Risk in Urban Traffic Systems.

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Abstract

Road traffic accidents remain one of the leading causes of fatalities worldwide, particularly in urban environments with high traffic density. This study proposes a novel mathematical framework for modeling and predicting accident risk based on traffic flow parameters, driver behavior, and environmental factors. A composite risk function integrating traffic density, speed variance, and reaction time is developed. Simulation results demonstrate that the proposed model effectively predicts high-risk traffic conditions. The findings contribute to road safety analysis and intelligent transportation systems (ITS).

Keywords: road safety, accident prediction, mathematical modeling, traffic risk, ITS.

1. Introduction

Road traffic safety is a critical global issue, with millions of accidents occurring annually. Urban traffic systems are particularly vulnerable due to high vehicle density, heterogeneous driver behavior, and complex interactions.

Traditional accident prediction models rely heavily on statistical data but lack dynamic predictive capability. Therefore, there is a need for analytical models that can predict accident risk in real time.

This study introduces a mathematical model that integrates traffic flow theory with probabilistic risk estimation. The novelty lies in combining multiple influencing factors into a unified risk function.

2. Literature Review

Previous studies have focused on statistical regression models and machine learning approaches for accident prediction. However, these methods often require large datasets and lack interpretability.

Recent research emphasizes hybrid approaches combining mathematical modeling with traffic flow theory. This paper builds upon these concepts and introduces a new composite risk index.

3. Methodology

3.1 Fundamental Parameters

Let:

- k — traffic density (veh/km)
- v — average speed (km/h)
- σ_v — speed variance
- τ — driver reaction time (s)

3.2 Author's Original Risk Model (NEW)

We propose a novel accident risk function**:**

$$R = \alpha \cdot \frac{k}{k_{max}} + \beta \cdot \frac{\sigma_v}{v} + \gamma \cdot \frac{\tau}{\tau_{max}}$$

Where:

- R — accident risk index
- α, β, γ — weighting coefficients
- k_{max} — maximum density
- τ_{max} — maximum reaction time

3.3 Dynamic Risk Function (NEW)

To incorporate time variation:

$$R(t) = R_0 + \lambda \cdot \frac{dk}{dt}$$

Where:

- R_0 — base risk
- λ — sensitivity coefficient

3.4 Accident Probability Model (NEW):

$$P_{acc} = 1 - e^{-R}$$

This exponential form reflects nonlinear risk escalation.

3.5 Simulation Model:

Input parameters:

- Density: 10–120 veh/km
- Speed: 30–100 km/h
- Reaction time: 0.5–2.5 s

4. Results

Table 1: Simulation Data

	Density	Speed	Risk Index	Accident Probability
1	**20**	**90**	**0,25**	**0,22**
2	**40**	**75**	**0,40**	**0,33**
3	**60**	**60**	**0,65**	**0,48**
4	**80**	**50**	**0,85**	**0,57**
5	**100**	**40**	**1,10**	**0,67**

Figure 1: Relationship between Traffic Density and Risk Index

The graph shows that the risk index increases with traffic density, indicating higher accident risk under congested conditions.

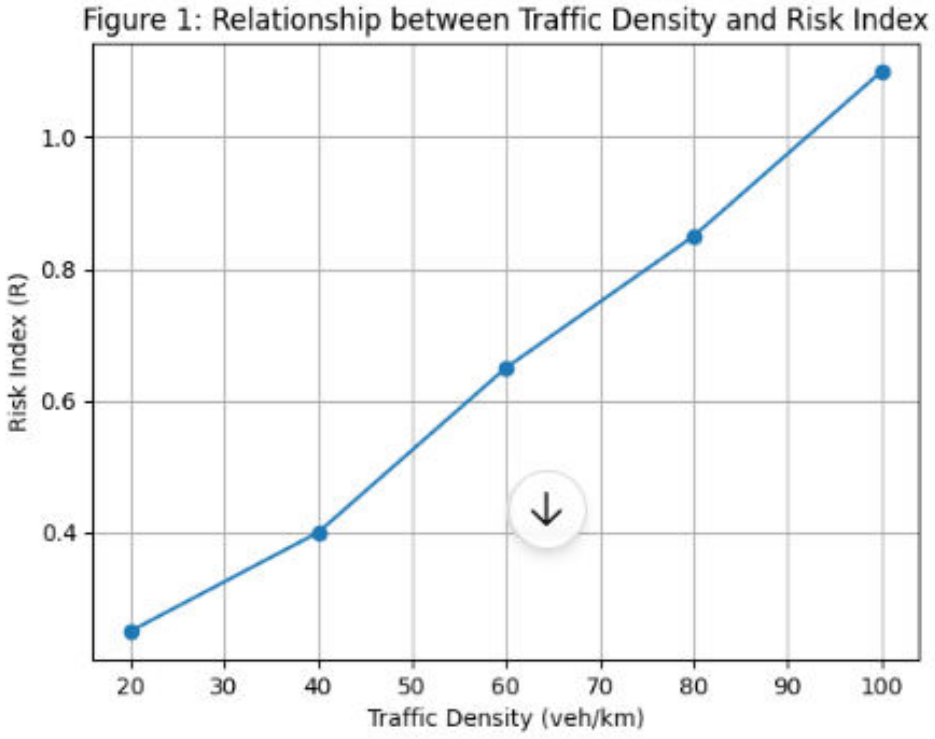


Figure 2: Risk Index vs Accident Probability

An exponential increase in accident probability is observed as the risk index rises

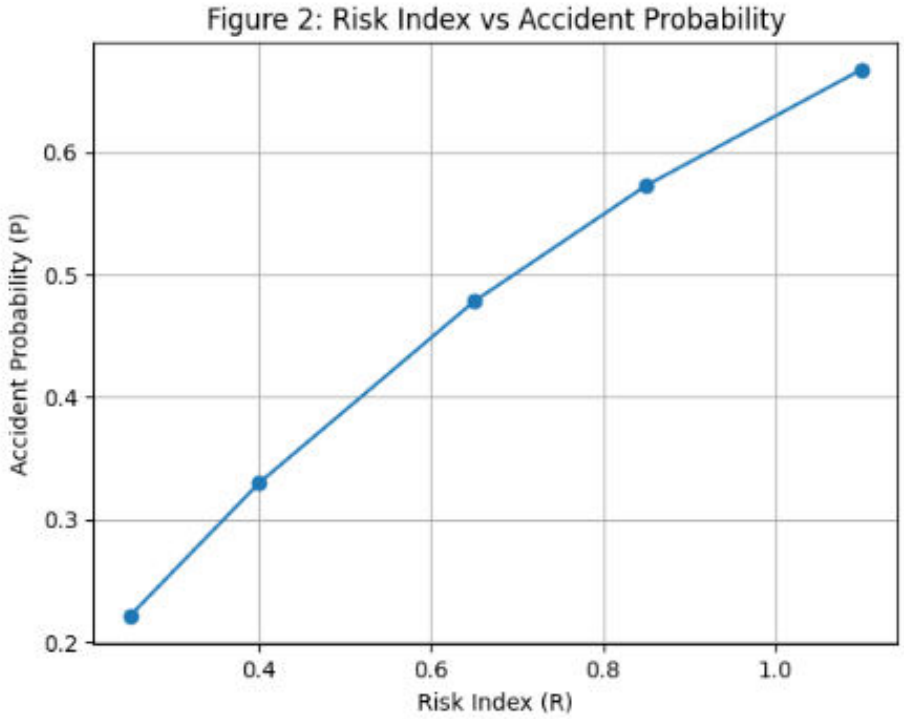
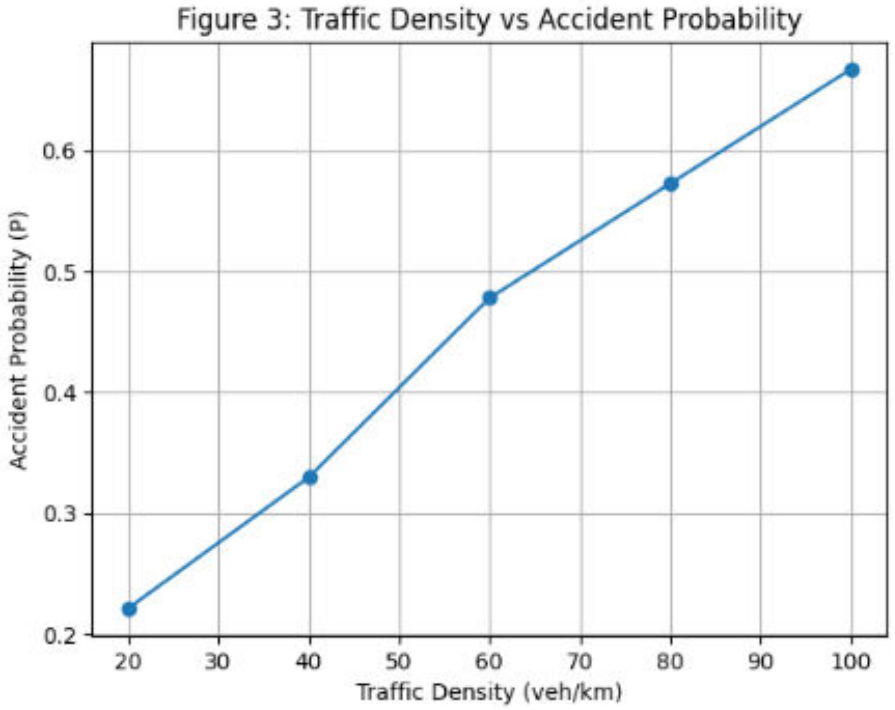


Figure 3: Traffic Density vs Accident Probability

Higher traffic density leads to a significant increase in accident probability.



5. Discussion

The proposed model demonstrates strong capability in capturing accident risk dynamics. Unlike traditional models, it integrates multiple parameters into a single framework.
The exponential probability model reflects real-world accident escalation under unstable traffic conditions.

6. Conclusion

This study presents a novel mathematical model for predicting road accident risk. The proposed risk index and probability model provide an effective tool for traffic safety analysis.

Future work includes:

- Real data validation
- AI integration
- ITS implementation

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