# NUMERICAL SIMULATION AND ACCURACY ASSESSMENT OF TRANSIENT FILTRATION IN POROUS MEDIA

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***Abstract.*** *This study investigates the influence of spatial mesh size and time step on the accuracy of numerical simulation of transient fluid filtration in porous media. The filtration process is described by a parabolic partial differential equation for pressure variation in space and time. An implicit finite difference scheme is employed, and the numerical results are compared with a reference analytical solution. Accuracy is evaluated using the root mean square error (RMSE) and maximum absolute error. The computations show that spatial mesh refinement substantially reduces the discretization error, while excessive reduction of the time step beyond a certain level increases the number of iterations without a comparable improvement in accuracy. The results demonstrate the need to select spatial and temporal discretization parameters jointly in order to balance accuracy and computational efficiency.*

***Keywords:*** *porous medium, transient filtration, finite difference method, spatial mesh size, time step, numerical accuracy, RMSE.*

## Introduction

Mathematical modeling of fluid filtration in porous media is important in petroleum reservoir engineering and groundwater studies. Uzbek researchers have developed numerical models and algorithms for oil and fluid filtration in porous media [1–3], while recent international studies continue to improve discretization and error-control techniques [4–6].

The accuracy of a numerical solution depends strongly on the spatial and temporal discretization. A coarse mesh may poorly resolve pressure gradients, whereas excessive refinement increases computational effort. This work evaluates the effects of the spatial mesh size $\Delta x$ and time step $\Delta t$ on a one-dimensional transient filtration model.

## Mathematical model and numerical method

For a one-dimensional homogeneous porous medium, transient single-phase filtration is described by

$$\varphi\, c_t \frac{\partial p}{\partial t} = \frac{\partial}{\partial x}\left[\frac{k}{\mu}\frac{\partial p}{\partial x}\right] \quad (1)$$

where $p = p(x,t)$ is pressure, $k$ is permeability, $\varphi$ is porosity, $\mu$ is dynamic viscosity, and $c_t$ is total compressibility. For constant medium properties, Eq. (1) reduces to the linear diffusivity equation

$$\frac{\partial p}{\partial t} = \alpha \frac{\partial^2 p}{\partial x^2}, \quad \alpha = \frac{k}{\varphi \mu c_t} \quad (2)$$

The initial and boundary conditions are given by

$$p(x,0) = p_0, \ 0 < x \le L; \quad p(0,t) = p_{in}, \quad p(L,t) = p_{out}, \ t > 0 \quad (3)$$

The inlet pressure is imposed for $t > 0$, corresponding to a step change at $t = 0^+$. In the numerical experiment, $L = 100$ m, $p_0 = 1$ MPa, $p_{in} = 2$ MPa, $p_{\text{out}} = 1$ MPa, $\alpha = 0.05$ m$^2$/s, and $T = 1000$ s. The domain is discretized by $x_i = i\Delta x$ and $t^n = n\Delta t$. Using backward Euler in time and a centered difference in space gives the implicit scheme; it is first-order accurate in time and second-order accurate in space.

$$\frac{p_i^{n+1} - p_i^n}{\Delta t} = \alpha \frac{p_{i+1}^{n+1} - 2p_i^{n+1} + p_{i-1}^{n+1}}{\Delta x^2} \quad (4)$$

At each time level, this gives the tridiagonal system

$$-r\, p_{i-1}^{n+1} + (1+2r) p_i^{n+1} - r\, p_{i+1}^{n+1} = p_i^n, \quad r = \frac{\alpha \Delta t}{\Delta x^2} \quad (5)$$

For the parameter values used in this study, with pressure expressed in MPa, the numerical solution is validated against the analytical series solution

$$p(x,t) = 2 - \frac{x}{L} - \sum_{n=1}^{\infty} \left[\frac{2}{n\pi}\right] \sin\left(\frac{n\pi x}{L}\right) \exp\left[-\alpha \left(\frac{n\pi}{L}\right)^2 t\right] \quad (6)$$

Accuracy is evaluated by the root mean square error (RMSE), where $N$ is the number of spatial grid nodes included in the error evaluation

$$RMSE = \sqrt{\frac{1}{N} \sum_{i=1}^{N} \left(p_i^{num} - p_i^{exact}\right)^2} \quad (7)$$

## Numerical results and discussion

First, $\Delta t = 20$ s was fixed and the spatial mesh was refined. The errors at $T = 1000$ s are given in Table 1.

*Table 1. Effect of spatial mesh size on numerical accuracy*

| $\Delta x$, m | Nodes | $\Delta t$, s | RMSE, kPa | Max. error, kPa |
|---|---|---|---|---|
| 10.0 | 11 | 20 | 7.722 | 23.141 |
| 5.0 | 21 | 20 | 2.543 | 6.988 |
| 2.5 | 41 | 20 | 1.099 | 3.237 |

Reducing $\Delta x$ from 10 m to 5 m decreases the RMSE from 7.722 kPa to 2.543 kPa. Further refinement to 2.5 m reduces the RMSE to 1.099 kPa. Thus, in this

test problem, spatial refinement produces a clear improvement in the numerical representation of the pressure field.

Next, $\Delta x = 5$ m was fixed and $\Delta t$ was varied (Table 2).

*Table 2. Effect of time step on numerical accuracy*

| $\Delta t$, s | Time steps | $\Delta x$, m | RMSE, kPa | Max. error, kPa |
|---|---|---|---|---|
| 40 | 25 | 5 | 2.992 | 7.752 |
| 20 | 50 | 5 | 2.543 | 6.988 |
| 10 | 100 | 5 | 2.373 | 6.773 |
| 5 | 200 | 5 | 2.305 | 6.664 |

Decreasing $\Delta t$ from 40 s to 20 s reduces the error. However, reducing $\Delta t$ from 10 s to 5 s doubles the number of time steps from 100 to 200, while RMSE improves only from 2.373 to 2.305 kPa.

The backward Euler scheme is unconditionally stable for this linear diffusion equation, but smaller $\Delta t$ increases computational cost; therefore, $\Delta x$ and $\Delta t$ should be chosen jointly for the required accuracy.

## Conclusion

The experiments show that both $\Delta x$ and $\Delta t$ affect transient filtration accuracy. Reducing $\Delta x$ from 10 m to 2.5 m lowered RMSE from 7.722 to 1.099 kPa, while the benefit of reducing $\Delta t$ became small below 10 s for $\Delta x = 5$ m. Thus, a balanced choice of spatial and temporal steps is preferable to excessive refinement of only one parameter. The approach can be extended to nonlinear and multidimensional filtration models with variable porous-medium properties.

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