THE RELATIONSHIP OF CONNECTIVITY BETWEEN CLASS OBJECTS

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Abstract. *The relationship of connectivity between class objects can be defined based on the internal properties of the objects, such as attributes or features, as well as their spatial arrangement or context. The higher the degree of connectivity, the more likely it is that the objects belong to the same class, and conversely, a low degree of connectivity may indicate that the objects belong to different classes.*

Machine learning algorithms and methods can be applied to assess and measure the relationship of connectivity between class objects based on available data or features. This can be useful for the automatic analysis of large volumes of data and for supporting decision-making based on the interrelations between class objects.

Keywords: *compactness, connectivity, claster, distance.*

Problem Statement

The formulation of the recognition problem is presented in a standard setting. For machine learning, a set of objects is given: $E_0=\{S_1,\ldots,S_m\}$, which is divided into $l(l>2)$**)** non overlapping subsets (classes) $K_1,\ldots,K_l$, such that $E_0=\bigcup_{i=1}^{l}K_i$. To identify the objects, a set of ***n* different features** is used: $X(n)=(x_1,\ldots,x_n)$, where ξ of the features are quantitative, and $(n\text{-}\xi)$ are nominal features. The similarity between objects in E_0 is calculated using a metric $\rho(x,y)$.

It is assumed that $L(E_0,\rho)$ is the subset of **boundary objects** of the classes, defined over E_0 using the metric $\rho(x,y)$. Objects $S_i,S_j\in K_t$, where $t=1,\ldots,l$, are considered **connected** to each other $(S_i\leftrightarrow S_j)$ if: $\{S\in L(E_0,\rho)|\rho(S,S_i)<r_i$ and $\rho(S,S_j)<r_j\}\neq\varnothing$, where $r_i(r_j)$ is the distance from $S_i(S_j)$ to the nearest object in CK_t ($CK_t=E_0\setminus K_t$) according to the metric $\rho(x,y)$. A subset $G_{tv}\subset K_t$, with $G_{tv}=\{S_{v_1},\ldots,S_{v_c}\}$, $c\geq 2$ and , $v<|K_t|$, is considered a **region (group) of connected objects** in class K_t if, for any $S_{v_i},S_{v_t}\in G_{tv}$, there exists a path $S_{v_i}\leftrightarrow S_{v_k}\leftrightarrow\ldots S_{v_l}$. An object $S_i\in K_t$, for $t=1,\ldots,l$, belongs to a **singleton group** and is considered **disconnected** if there exists

no path $S_i \leftrightarrow S_j$ for any object $S_j \neq S_i$, where $S_j \in K_t$. The task is to determine the **minimal covering** consisting of **non-overlapping groups** based on the **connectivity relation** of the objects for each class K_t, where $t = 1,\ldots,l$.

When determining the minimal number of groups of connected and unconnected objects of classes, $L(E_0,\rho)$ is used – a subset of boundary objects (the shell) of the classes based on a given metric ρ and the description of objects in a new space composed of binary features. To extract the class shell for each $S_i \in K_t$, $t = 1,...,l$, an ordered sequence based on $\rho(x, y)$ is constructed.

$$S_{i_0}, S_{i_1}, ..., S_{i_{m-1}}, S_i = S_{i_0}. \quad (1)$$

Let the nearest object to Si from (1), which does not belong to class K_t, be considered. Denote by $O(S_i)$ the neighborhood of radius $r_i = \rho(S_i, S_{i_\beta})$ centered at S_i, including all objects for which $\rho(S_i, S_{i_\tau}) < r_i$, $\tau = 1,\ldots,\beta$-1. In $O(S_i)$, there always exists a non-empty subset of objects

$$\Delta_i = \left\{ S_{i_\alpha} \in O(S_i) \mid \rho(S_{i_\beta}, S_{i_\alpha}) = \min_{S_{i_\tau} \in O(S_i)} \rho(S_{i_\beta}, S_{i_\tau}) \right\} \quad (2)$$

According to (1.6), the membership of objects in the class shell is defined as $L(E_0,\rho) = \bigcup_{i=1}^{m} \Delta_i$.

The set of shell objects from Kt ∩ L(E₀, ρ) is denoted as $L_t(E_0,\rho) = \{S^1,\ldots,S^\pi\}$, $\pi \geq 1$, where $\pi = 1$. The value $\pi = 1$ uniquely determines that all objects of the class belong to a single group. When $\pi \geq 2$, the description of each object $S_i \in K_t$ is transformed into $S_i = (y_{i1},...,y_{i\pi})$, where

$$y_{ij} = \begin{cases} 1, \rho(S_i, S^j) < r_i, \\ 0, \rho(S_i, S^j) \geq r_i. \end{cases} \quad (3)$$

Let, according to (3), a description of the objects of class K_t in the new (binary) feature space be obtained, where $\Omega = K_t$, θ is the number of mutually disjoint groups of objects, $S_\mu \vee S_\eta$, $S_\mu \wedge S_\eta$ – are, respectively, the operations of disjunction and conjunction over the binary features of the objects $S_\mu, S_\eta \in K_t$. A step-by-step execution of the algorithm for partitioning the objects of K_t into non-overlapping groups $G_1,\ldots,G_\theta$ is as follows.

Step 1: θ=0;

Step 2: Identify the object $S \in \Omega$, $\theta = \theta + 1$, $Z = S$, $G_\theta = \varnothing$;

Step 3: Implement selection $S \in \Omega$ и $S \wedge Z = true$, $\Omega = \Omega \backslash S$, $G_\theta = G_\theta \cup S$, $Z = Z \vee S$ while $\{S \in \Omega \mid S \wedge Z = true\} \neq \varnothing$;

Step 4: If $\Omega \neq \varnothing$, then go to 2;

Step 5: Print results.

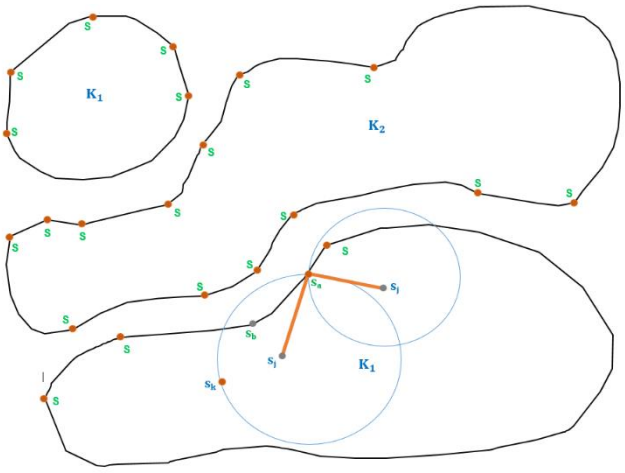


Figure 1.1. Connectivity of objects of class K1 via a chain $S_i \leftrightarrow S_k \leftrightarrow ... \leftrightarrow S_c$.

The properties of the object connectivity relation are as follows:

- The sample E0 is divided into a unique and fixed number of non-overlapping groups of objects;
- Between any two objects Si and Sj from the same group, it is always possible to construct a chain $S_i \leftrightarrow S_k \leftrightarrow ... \leftrightarrow S_c$.

A graphical representation of connectivity via a chain of intersecting hyperspheres is shown in Figure 1.1.

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