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EFFECT OF CROSS-SECTIONAL HEIGHT ON THE NATURAL FREQUENCIES OF A CANTILEVER BEAM: AN ANSYS-BASED INVESTIGATION

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Abstract. *The dynamic characteristics of beam structures are strongly influenced by their geometric and material parameters. In this study, the effect of cross-sectional height on the natural frequencies of a rectangular cantilever beam is investigated using the finite element method implemented in ANSYS. A three-dimensional cantilever beam with a length of 140 mm and a width of 25 mm was considered, while the cross-sectional height was varied as 10, 12.5, and 15 mm. Structural steel with a Young's modulus of 207 GPa, Poisson's ratio of 0.30, and density of 8050 kg/m³ was used. Modal analysis was performed to determine the natural frequencies and corresponding vibration modes. The first six natural frequencies were obtained for each cross-sectional configuration. In addition, the model with a cross-sectional height of 12.5 mm was compared with previously reported numerical results to verify the developed finite element model. The results demonstrate that changes in cross-sectional height affect different vibration modes to different extents.*

Keywords*: Cantilever beam; Modal analysis; Natural frequency; Finite element method; ANSYS; Cross-sectional height; Vibration modes.*

1. INTRODUCTION.

Beams are fundamental structural elements widely used in civil and mechanical engineering structures. Their vibration characteristics are important in structural analysis and design because natural frequencies and mode shapes provide essential information about the dynamic behavior of beam structures. Previous research has investigated the free vibration characteristics of beams under different geometric and boundary conditions using analytical and numerical approaches [1,2].

The natural frequencies of beam structures depend on several parameters, including material properties, boundary conditions, beam dimensions, and cross-sectional geometry. In particular, changes in the cross-sectional dimensions affect the stiffness and mass characteristics of a beam and consequently influence its natural

frequencies. The influence of geometric characteristics and boundary conditions on beam vibration has been investigated in previous studies [2].

Analytical beam theories provide an effective approach for determining natural frequencies of beams. The Euler–Bernoulli beam theory is one of the classical approaches used for the analysis of beam bending and vibration problems [3]. General treatments of mechanical vibration also provide the theoretical basis for evaluating natural frequencies and vibration modes of structural systems [4]. However, numerical methods such as the finite element method allow detailed investigation of beam structures and their vibration modes, and their applicability to modal analysis has been demonstrated in previous research [1].

In the present study, the influence of cross-sectional height on the natural frequencies of a rectangular cantilever beam is investigated using finite element modal analysis in ANSYS. A three-dimensional beam model with a constant length and width is considered, while the cross-sectional height is varied between 10, 12.5, and 15 mm. The first six natural frequencies are determined for each configuration. In addition, the model with a cross-sectional height of 12.5 mm is compared with previously reported numerical results to verify the consistency of the developed finite element model.

The aim of this study is to evaluate the effect of cross-sectional height on the natural frequencies of a rectangular cantilever beam and to identify the sensitivity of different vibration modes to changes in cross-sectional geometry.

2. NUMERICAL MODEL AND METHODOLOGY.

A three-dimensional rectangular cantilever beam was investigated using the finite element method in ANSYS. The beam was modeled with a fixed-free boundary condition. The length and width were kept constant at $L = 140\text{mm}$ and $b = 25\text{mm}$, while the cross-sectional height was varied as $h = 10, 12.5,$ and 15mm. The geometric parameters of the investigated models are presented in Table 1.

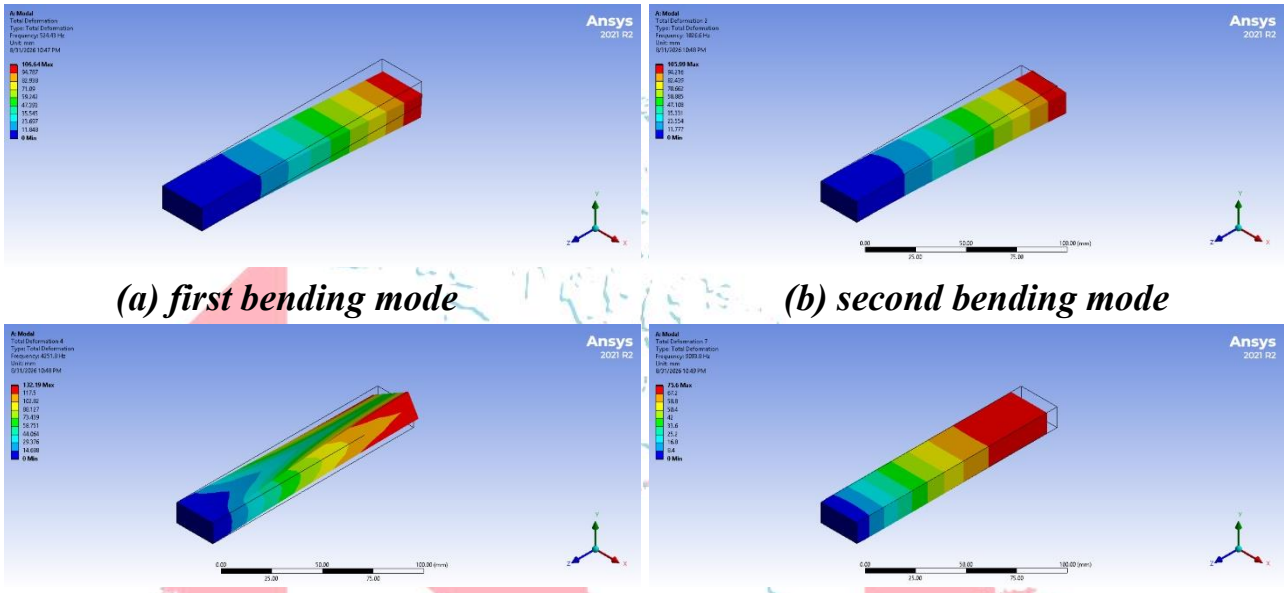
Table 1. Geometric parameters of the investigated beam models

Parameter	Case 1	Case 2	Case 3
Length, *L*, mm	140	140	140
Width, *b*, mm	25	25	25
Height, *h*, mm	10	12.5	15

Structural steel was used for all models, with a Young's modulus of $E = 207\text{GPa}$, Poisson's ratio of $\nu = 0.30$, and density of $\rho = 8050\text{kg/m}^3$. These material properties were kept constant for all configurations in order to evaluate only the effect of the cross-sectional height.

2.1. Finite Element Model. The three-dimensional beam models were generated in ANSYS 2021 R2. A fixed support was applied to one end of the beam, while the opposite end was left free. The same modeling procedure and boundary conditions were used for all three configurations.

A modal analysis was performed to determine the natural frequencies and corresponding mode shapes of the beam. The first six eigenfrequencies were extracted for each cross-sectional height and used for comparison. The numerical analysis was performed using the finite element procedures implemented in ANSYS [5].



(a) first bending mode *(b) second bending mode*

(c) torsional mode *(d) longitudinal mode*

Figure 1. Representative vibration modes of the cantilever beam obtained from ANSYS

3. RESULTS AND DISCUSSION.

3.1. Effect of Cross-Sectional Height.

Table 2. Natural frequencies for different cross-sectional heights

Mode	Vibration type	$h = 10$ mm	$h = 12.5$ mm	$h = 15$ mm
1	Out-of-plane bending	422.12	524.43	627.56
2	In-plane bending	1026.1	1026.6	1027.3
3	Out-of-plane bending	2590.1	3172.4	3741.7
4	Torsion	3767.5	4251.8	4644.5
5	In-plane bending	5683.3	5685.7	5688.4
6	Out-of-plane bending	7033.0	8449.3	9096.8

The numerical results presented in Table 2 demonstrate that the influence of cross-sectional height on the natural frequencies depends strongly on the vibration mode.

When the height of the beam increases from 10 mm to 15 mm, the first natural frequency increases from 422.12 Hz to 627.56 Hz, corresponding to an increase of approximately 48.7%. The third, fourth, and sixth natural frequencies also increase from 2590.1 Hz to 3741.7 Hz, from 3767.5 Hz to 4644.5 Hz, and from 7033.0 Hz to 9096.8 Hz, respectively.

In contrast, the second and fifth modes exhibit almost unchanged natural frequencies. The second-mode frequency changes only from 1026.1 Hz to 1027.3 Hz, while the fifth-mode frequency changes from 5683.3 Hz to 5688.4 Hz. These results indicate that the effect of cross-sectional height is strongly dependent on the vibration mode. The geometric modification does not produce a uniform shift in all natural frequencies, but affects the modal characteristics according to the corresponding deformation pattern.

3.2. Comparison with Previously Reported Results

Table 3.

Mode	Vibration type	Fadalla, Hz [1]	ANSYS, Hz	Difference, %
1	Out-of-plane bending	523.98	524.43	0.086
2	In-plane bending	1026.3	1026.6	0.029
3	Out-of-plane bending	3173.7	3172.4	0.041
4	Torsion	4260.0	4251.8	0.192
5	In-plane bending	5689.3	5685.7	0.063
6	Out-of-plane bending	8472.9	8449.3	0.279

To verify the consistency of the developed finite element model, the results obtained for the reference beam with $h = 12.5$ mm were compared with the numerical results reported by Fadalla et al. As shown in Table 3, very close agreement is obtained for all of the first six modes. The maximum difference is 0.279%, while the minimum difference is only 0.029%. The close agreement confirms that the present ANSYS model provides results consistent with the previously reported numerical solution and provides a reliable basis for the subsequent investigation of the effect of cross-sectional height.

4. CONCLUSION.

In this study, the influence of cross-sectional height on the natural frequencies of a rectangular cantilever beam was investigated using finite element modal analysis in ANSYS. A three-dimensional beam with a length of 140 mm and a width of 25 mm was considered, while the cross-sectional height was varied as 10, 12.5, and 15 mm. The results showed that increasing the cross-sectional height significantly affects some

vibration modes, while others remain almost unchanged. In particular, the first natural frequency increased by approximately 48.7% when the height increased from 10 mm to 15 mm, whereas the second and fifth modes exhibited negligible changes.

The reference configuration with $h = 12.5$ mm was compared with previously reported ANSYS results, and the differences for the first six modes were less than 0.3%, confirming the consistency of the developed numerical model. The obtained results demonstrate that the effect of cross-sectional geometry on beam vibration characteristics is mode-dependent and should be considered in the dynamic analysis and design of beam-type structural elements.

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