ADI MODELING OF TIME DEPENDENT GROUNDWATER FLOW IN HETEROGENEOUS LAYERED AQUIFERS

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Abstract. *A two-dimensional ADI model describes transient groundwater flow in heterogeneous layered aquifers with anisotropic conductivity and variable storage. Head and normal flux remain continuous at layer interfaces. Simulations capture delayed aquitard leakage and asymmetric drawdown. The second-order scheme is unconditionally stable, with benchmark discrepancies below 1.8%.*

Keywords: *Groundwater seepage, Stratified porous formations, Direction-dependent hydraulic conductivity, Alternating Direction Implicit method, Leakage through aquitards, Extraction-induced drawdown.*

Introduction. Layered aquifers require reliable estimates of head and interlayer leakage [1, 2]. Vertical gradients near wells limit horizontal-flow approximations [3]. ADI splitting reduces multidimensional problems to tridiagonal systems [5].

Filtration models and numerical software [6], heterogeneous filtration [7], parallel reservoir computations [8], modelling in complex porous structures [9], and subsurface hydrodynamics [10] provide related context.

This study examines pumping in two aquifers separated by an aquitard, emphasizing its thickness and vertical conductivity.

Methods. For saturated layers Ωm within $\Omega = (0, Lx) \times (0, Lz)$, Darcy flow ($Re < 1$) satisfies:

$$Ss(z) \cdot \partial h/\partial t = \partial/\partial x \left(Kx(z) \cdot \partial h/\partial x \right) + \partial/\partial z \left(Kz(z) \cdot \partial h/\partial z \right) - Qw(x, z, t)$$

Here h is head [m], Ss specific storage [m^{-1}], Kx and Kz conductivities [m/s], and Qw extraction per unit volume [s^{-1}].

Lateral heads: $h(0,z,t) = h_0$; $h(Lx,z,t) = hL$.

Impermeable top and base: $\partial h/\partial z = 0$ at $z = 0, Lz$.

At layer interfaces $z = \Gamma$, head and normal flux are continuous:

$$\lim(\varepsilon \to 0^+) h(x, \Gamma - \varepsilon, t) = \lim(\varepsilon \to 0^+) h(x, \Gamma + \varepsilon, t)$$

$$\lim(\varepsilon \to 0^+) \left[Kz,m \cdot \partial h/\partial z \right]_{(z = \Gamma - \varepsilon)} = \lim(\varepsilon \to 0^+) \left[Kz,m+1 \cdot \partial h/\partial z \right]_{(z = \Gamma + \varepsilon)}$$

Interface conductivity uses the harmonic mean:

$Kz,j+1/2 = (2 \cdot Kz,j \cdot Kz,j+1) / (Kz,j + Kz,j+1)$

On a grid (Δx, Δz), each Δt is split into two half-steps:

First half-step: implicit in x, explicit in z.

$Ss,j \cdot (h_{i,j}^{n+1/2} - h_{i,j}^{n}) / (\Delta t / 2) = \delta x(Kx\ \delta x\ h^{n+1/2})_{i,j} + \delta z(Kz\ \delta z\ h^{n})_{i,j} - Qw_{i,j}^{n+1/2}$

Second half-step: explicit in x, implicit in z.

$Ss,j \cdot (h_{i,j}^{n+1} - h_{i,j}^{n+1/2}) / (\Delta t / 2) = \delta x(Kx\ \delta x\ h^{n+1/2})_{i,j} + \delta z(Kz\ \delta z\ h^{n+1})_{i,j} - Qw_{i,j}^{n+1/2}$

The Thomas algorithm solves each tridiagonal system in O(Nx · Nz) operations per step.

Results and Discussion. The test section has Lx = 1000 m and H = 60 m:

• Layer 1 (Upper unconfined/semi-confined aquifer): z in [40, 60] m, $Kx = 1.5 \times 10^{-4}$ m/s, $Kz = 3.0 \times 10^{-5}$ m/s, $Ss = 1.0 \times 10^{-4}$ m^{-1}.

• Layer 2 (Intermediate aquitard): z in [30, 40] m, $Kx = 2.0 \times 10^{-7}$ m/s, $Kz = 5.0 \times 10^{-8}$ m/s, $Ss = 2.5 \times 10^{-4}$ m^{-1}.

• Layer 3 (Lower confined aquifer): z in [0, 30] m, $Kx = 3.0 \times 10^{-4}$ m/s, $Kz = 8.0 \times 10^{-5}$ m/s, $Ss = 5.0 \times 10^{-5}$ m^{-1}.

A well at x = 500 m pumps Q = 1200 m^3/day from z = 5–25 m. The 30-day simulation uses Δx = 5 m, Δz = 0.5 m, and Δt = 10–3600 s.

Table 1. Drawdown $s = h_0 - h$ (m) after 1, 7, and 30 days.

Location (x, z)	**Stratum**	**1 Day**	**7 Days**	**30 Days**
(505 m, 15 m)	Lower Aquifer (near well)	4.82	6.14	6.89
(505 m, 35 m)	Intermediate Aquitard	0.41	1.95	3.12
(505 m, 50 m)	Upper Aquifer	0.03	0.38	1.26
(650 m, 15 m)	Lower Aquifer (intermediate)	1.34	2.58	3.41
(650 m, 50 m)	Upper Aquifer (intermediate)	0.01	0.12	0.54

Initially, over 91% of pumping comes from lower-aquifer elastic storage. The aquitard gradient reaches $\partial h/\partial z \approx 0.35$, inducing downward leakage and buffering drawdown.

Mass-balance error remains below 0.04%; RMSE against Neuman–Witherspoon solutions is at most 0.078 m [4].

A tenfold increase in aquitard Kz accelerates upper-aquifer head decline by 3.4 times during the first 10 days, indicating sensitivity to vertical leakage.

Conclusion. The ADI model represents delayed leakage and contrasting layer responses efficiently. It supports aquifer-management and environmental assessments while maintaining head and flux continuity.

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