# FINITE VOLUME SIMULATION OF COUPLED GROUNDWATER FLOW AND SOLUTE TRANSPORT THROUGH HIGHLY HETEROGENEOUS POROUS LAYERS

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***Abstract.*** *A cell-centered finite volume model couples groundwater flow and non-reactive solute transport in heterogeneous layers. Conservative interface fluxes and a TVD limiter reduce numerical diffusion and oscillations. Simulations show preferential migration in permeable strata and retention followed by back-diffusion in low-permeability layers, extending remediation times.*

***Keywords:*** *Stratified porous formations, Dissolved-species transport, Advection-dispersion model, Finite volume approach, Total Variation Diminishing flux limiter, Interface monitoring.*

**Introduction.** Layered aquifers are vulnerable to dissolved contaminants [1, 5]. Preferential advection and diffusion into low-permeability beds produce delayed release and plume tailing [4, 5]. Central differences may oscillate when $\mathrm{Pe} > 2$, while first-order upwinding introduces numerical diffusion [2, 3].

Heterogeneous filtration [6], reservoir simulation [7], and non-Fickian, fractional, and double-relaxation transport [8–10] provide related context.

This study applies a conservative finite volume scheme with a TVD limiter to coupled flow and transport.

**Methods.** Steady groundwater flow obeys:

$$\nabla \cdot q = 0, \quad q = -K(x,z)\nabla h$$

Here $q = (u,w)^{\mathrm{T}}$ is Darcy flux [m/s], $K$ conductivity, $h$ head [m], $\theta$ porosity, and $v = q/\theta$ pore velocity.

Non-reactive solute concentration $C$ obeys:

$$\frac{\partial(\theta C)}{\partial t} + \nabla \cdot (qC) = \nabla \cdot (\theta D \nabla C)$$

$C$ is measured in kg/m$^3$ or mg/L. The dispersion tensor $D$ is:

$$D_{xx} = \alpha_{\mathrm{L}} \cdot \left(\frac{u^2}{|v|}\right) + \alpha_{\mathrm{T}} \cdot \left(\frac{w^2}{|v|}\right) + D_{\mathrm{m}}^{*}$$

$$D_{zz} = \alpha_{\mathrm{T}} \cdot \left(\frac{u^2}{|v|}\right) + \alpha_{\mathrm{L}} \cdot \left(\frac{w^2}{|v|}\right) + D_{\mathrm{m}}^*$$

$$D_{xz} = D_{zx} = (\alpha_{\mathrm{L}} - \alpha_{\mathrm{T}}) \cdot \left(\frac{u \cdot w}{|v|}\right)$$

$\alpha_{\mathrm{L}}$ and $\alpha_{\mathrm{T}}$ are dispersivities [m]; $D_{\mathrm{m}}^*$ is effective molecular diffusion [$\mathrm{m}^2/\mathrm{s}$].

Integration over $V_{i,j} = \Delta x \Delta z$ gives:

$$\theta_{i,j} \cdot \frac{\left(C_{i,j}^{n+1} - C_{i,j}^{n}\right)}{\Delta t} \cdot \Delta x \Delta z + \sum_{f \in \{\mathrm{e,w,n,s}\}} \left(F_{\mathrm{adv},f} - F_{\mathrm{diff},f}\right) = 0$$

Face fluxes are $F_{\mathrm{adv},f} = (q \cdot n)_f C_f$ and $F_{\mathrm{diff},f} = (\theta D \nabla C \cdot n)_f A_f$. The Superbee reconstruction is:

$$C_{i+1/2,j} = C_U + 0.5 \cdot \varphi(r) \cdot (C_D - C_U)$$

For upwind $U$ and downwind $D$ cells:

$$r = \frac{(C_U - C_{U-1})}{(C_D - C_U)}, \quad \varphi(r) = \max(0, \min(2r, 1), \min(r, 2))$$

Harmonic transmission conditions conserve diffusive flux at layer interfaces.

**Results and Discussion.** The domain ($L = 200$ m, $H = 20$ m) contains:

• Strata A ($z = 12$ to 20 m): Coarse sand ($K_1 = 5.0 \times 10^{-4}$ m/s, $\theta_1 = 0.32$, $\alpha_{\mathrm{L}} = 1.0$ m, $\alpha_{\mathrm{T}} = 0.1$ m).

• Strata B ($z = 8$ to 12 m): Silt-clay lens ($K_2 = 2.0 \times 10^{-7}$ m/s, $\theta_2 = 0.45$, $\alpha_{\mathrm{L}} = 0.1$ m, $\alpha_{\mathrm{T}} = 0.01$ m, $D_{\mathrm{m}}^* = 1.2 \times 10^{-9}$ $\mathrm{m}^2/\mathrm{s}$).

• Strata C ($z = 0$ to 8 m): Medium sand ($K_3 = 1.2 \times 10^{-4}$ m/s, $\theta_3 = 0.35$, $\alpha_{\mathrm{L}} = 0.8$ m, $\alpha_{\mathrm{T}} = 0.08$ m).

Heads: $h(0, z) = 22$ m; $h(L, z) = 20$ m; $J = 0.01$.

At $x = 0$, $z = 10$–18 m, $C_0 = 100$ mg/L is injected for 15 days, followed by clean-water flushing ($C = 0$).

Stratum A transports solute at $v_x \approx 1.56 \times 10^{-5}$ m/s (1.35 m/day), reaching $x = 200$ m in about 145 days. Stratum B has $v_x \approx 0.0004$ m/day; diffusion dominates its uptake.

Table 2. Concentration (mg/L) at $x = 100$ m across the layers.

| **Time (Days)** | **Strata A** ($z = 15$ m) | **Strata B** ($z = 10$ m) | **Strata C** ($z = 5$ m) |
|---|---|---|---|
| 10 | 0.00 | 0.00 | 0.00 |
| 30 | 12.45 | 0.02 | 0.00 |
| 60 | 78.10 | 1.84 | 0.15 |
| 90 (Flushing) | 18.20 | 6.72 | 1.82 |
| 150 (Flushing) | 1.15 | 4.85 | 1.20 |
| 300 (Tail) | 0.08 | 1.42 | 0.34 |

After $t > 90$ days, upper-aquifer concentrations fall, but back-diffusion from Stratum B sustains tailing at $C \approx 0.08–1.15$ mg/L.

The TVD limiter avoids oscillations at $\mathrm{Pe} = 18.2$. First-order upwinding overestimates aquitard penetration by 42%.

Low-permeability beds prolong cleanup; their characteristic diffusion timescale is:

$$\tau_{\mathrm{diff}} \approx \frac{b^2}{(4 \cdot D_{\mathrm{m}}^*)}$$

For $b = 4$ m, $\tau_{\mathrm{diff}}$ exceeds several years.

**Conclusion.** Conservative finite volume discretization with TVD limiting resolves transport across permeability contrasts. Matrix storage and back-diffusion must be included in remediation estimates.

## References

1. Chapelle, F. H. (2001). Groundwater Microbiology and Geochemistry. John Wiley & Sons, New York.
2. Harten, A. (1983). High resolution schemes for hyperbolic conservation laws. Journal of Computational Physics, 49(3), 357–393.
3. LeVeque, R. J. (2002). Finite Volume Methods for Hyperbolic Problems. Cambridge University Press, Cambridge.
4. Sudicky, E. A., and Frind, E. O. (1982). Contaminant transport in fractured porous media: Analytical solutions for a system of parallel fractures. Water Resources Research, 18(6), 1634–1642.
5. Zheng, C., and Bennett, G. D. (2002). Applied Contaminant Transport Modeling. 2nd Edition, John Wiley & Sons, New York.
6. Хужаёров Б. Х., Махмудов Ж. М. (2014). Математические модели фильтрации неоднородных жидкостей в пористых средах. Ташкент: Фан.
7. Aziz, K., and Settari, A. (1979). Petroleum Reservoir Simulation. London: Applied Science Publishers.
8. Fomin, S. A., Chugunov, V. A., and Hashida, T. (2011). Non-Fickian mass transport in fractured porous media. Advances in Water Resources, 34(2), 205–214.
9. Suzuki, A., Horne, R. N., Makita, H., Niibori, Y., Fomin, S. A., Chugunov, V. A., and Hashida, T. (2013). Development of fractional derivative-based mass and heat transport model. Proceedings of the Thirty-Eighth Workshop on Geothermal Reservoir Engineering, Stanford University, February 11–13, 2013. SGP-TR-198.
10. Khuzhayorov, B., Dzhiyanov, T., and Khaydarov, O. (2018). Double-Relaxation Solute Transport in Porous Media. International Journal of Advanced Research in Science, Engineering and Technology, 5(1), 5094–5100.